MATH 208 C **Midterm 1** **Spring 2022**

NAME:

UW ID: | | | | | | | |

Academic Honesty Statement: All work on this exam is my own.

Signature:

INSTRUCTIONS

1. This exam contains eight (8) printed pages. Check that on your exam the bottom of the last page says **END OF EXAM**.
2. The points for each question are indicated at the beginning of each question.
3. **Show all your work**, unless the problem says otherwise explicitly. An answer without work shown will receive little or no credit.
4. If you need more space for your answer, use the back of the page and indicate that you have done so.
5. You may use one sheet of handwritten notes on a 8.5×11 inch paper, both sides. You may use a TI-30X IIS calculator. No other resources are allowed.
6. Raise your hand if you have a question.
7. Time allowed: 50 minutes.

1	/10
2	/25
3	/10
4	/10
5	/10
6	/10
7	/15
Total	/90

(1) [5 points each] Compute:

(i) $8 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 60 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 100 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} =$

(ii) $\begin{bmatrix} 4 & 0 & -2 & 1 \\ 3 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \\ 5 \end{bmatrix} =$

(2) (a) [10 points] Determine the general solution to the following system of equations:

$$\begin{array}{rcrcrcr} w_1 & + & w_2 & + & 2w_3 & = & 1 \\ 3w_1 & + & 3w_2 & + & 7w_3 & = & -1 \end{array} .$$

Show all your work.

(b) [5 points] Find an expression of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix}$. Give an answer with constants, using no variables or parameters.
(Hint: use part (a).)

(c) [10 points] Determine all values of a and b such that the following system of equations in w_1, w_2, w_3 has no solutions.

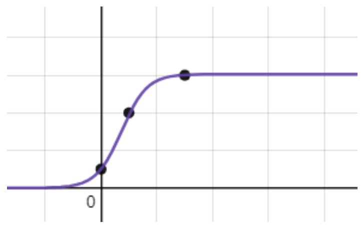
$$\begin{array}{rcrcrcl} w_1 & + & w_2 & + & 2w_3 & = & 1 \\ 3w_1 & + & 3w_2 & + & aw_3 & = & b \end{array} .$$

Show all your work.

(3) A *logistic curve* is a curve in the (x, y)-plane defined by an equation of the form

$$y(1 + ae^{-x}) = b,$$

where a and b are constants. Suppose we want a logistic curve that passes through the points $(0, 1)$, $(1, 4)$, and $(3, 6)$. How can we find its equation?



(a) [5 points] Write a system of linear equations in a, b to describe the coefficients of a logistic curve which passes through the three given points. (Write out equations with variables.) Do not simplify constants of the form e^{-c}. You do not have to solve the system. Show your work, and put a **box around your answer**.

(b) [5 points] Write the augmented matrix of the system found in part (a). You do not have to solve the system.

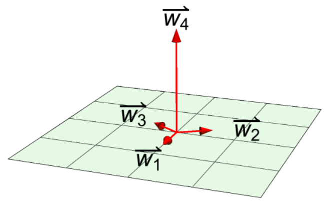
(4) (a) [5 points] Does this set of vectors span $\mathbb{R}^5$? Justify your answer.

$$\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 3 \\ 9 \\ 0 \\ 1 \end{bmatrix}, \ \vec{\mathbf{v}}_2 = \begin{bmatrix} 3 \\ 0 \\ 7 \\ 7 \\ 1 \end{bmatrix}, \ \vec{\mathbf{v}}_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

(b) [5 points] Does this set of vectors span $\mathbb{R}^4$? Justify your answer.

$$\vec{\mathbf{u}}_1 = \begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \end{bmatrix}, \ \vec{\mathbf{u}}_2 = \begin{bmatrix} 1 \\ 1 \\ -3 \\ 1 \end{bmatrix}, \ \vec{\mathbf{u}}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -3 \end{bmatrix}, \ \vec{\mathbf{u}}_4 = \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

(5) [10 points] Consider the vectors $\vec{\mathbf{w}}_1, \ldots, \vec{\mathbf{w}}_4$ as shown below. (The plane contains $\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3$.)



(a) [5 points] Which of the following sets of vectors are linearly independent? Circle all that apply. No justification is needed.

$\{\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2\}$ $\{\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_4\}$ $\{\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_4\}$

$\{\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3\}$ $\{\vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3, \vec{\mathbf{w}}_4\}$ $\{\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3, \vec{\mathbf{w}}_4\}$

(b) [5 points] Consider the linear transformation $S : \mathbb{R}^4 \to \mathbb{R}^3$ defined by

$$S\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = x_1\vec{\mathbf{w}}_1 + x_2\vec{\mathbf{w}}_2 + x_3\vec{\mathbf{w}}_3 + x_4\vec{\mathbf{w}}_4.$$

Is S one-to-one? Explain why or why not. You may use part (a) in your explanation.

(6) [10 points] Suppose $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{u}}, \vec{\mathbf{w}}$ are vectors in $\mathbb{R}^n$ such that

- $\vec{\mathbf{u}}$ is in span$\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2\}$, and
- $\vec{\mathbf{w}}$ is not in span$\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2\}$.

Is $\vec{\mathbf{u}} + \vec{\mathbf{w}}$ in span$\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2\}$? Justify your answer.

(Hint: Consider the vector diagram in problem (5) on the previous page.)

(7) [5 points each] In each of the following, either give an example or write "not possible". No justification is necessary.

(a) A linear system in echelon form whose general solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

(b) A set of vectors in $\mathbb{R}^4$ that is dependent, and spans $\mathbb{R}^4$.

(c) Vectors $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3$ in $\mathbb{R}^3$ such that $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2\}$ is dependent, and $\text{span}\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\} = \mathbb{R}^3$.

END OF EXAM